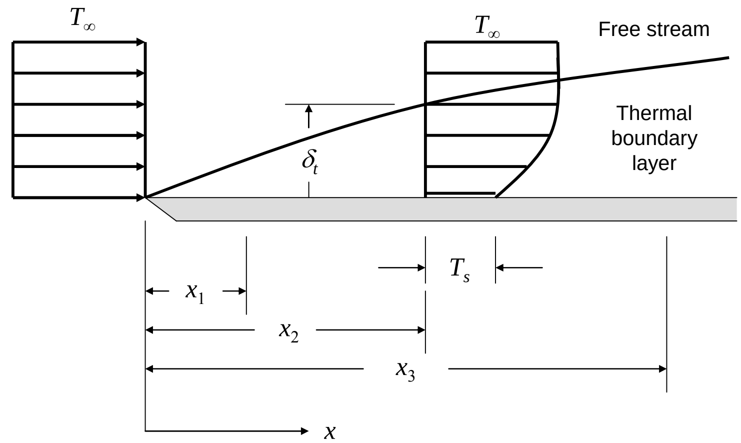
PROBLEM 01

KNOWN: Temperature distribution at x_2 in laminar thermal boundary layer.
FIND: (a) Whether plate is being heated or cooled, (b) Temperature distributions at two other x locations. Locations of largest and smallest heat fluxes, (c) Temperature distribution at x_2 for lower and higher free stream velocities. Which velocity condition causes the largest heat flux.

SCHEMATIC:



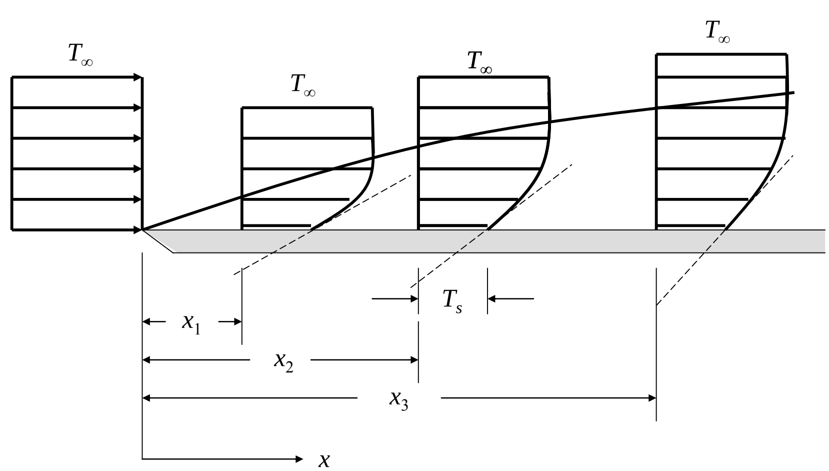
ASSUMPTIONS: (1) Steady-state conditions, (2) Laminar, incompressible flow.

ANALYSIS: (a) Since the sketch indicates that the free stream temperature is greater than the surface temperature, the plate is being heated by the fluid. This is consistent with the fact that the surface heat flux in the positive y-direction is given by Eq. 6.3:

$$q_s'' = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

From the sketch, the temperature gradient is positive, therefore the heat flux is negative. The heat transfer is in the negative y-direction, the plate is being heated by the fluid. <

(b) At every location in the boundary layer, the temperature must vary from T_s at the surface to T_∞ in the free stream. This change must occur within the thermal boundary layer thickness, as shown in the sketch below.



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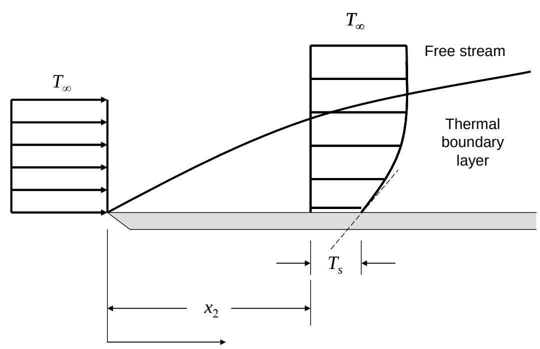
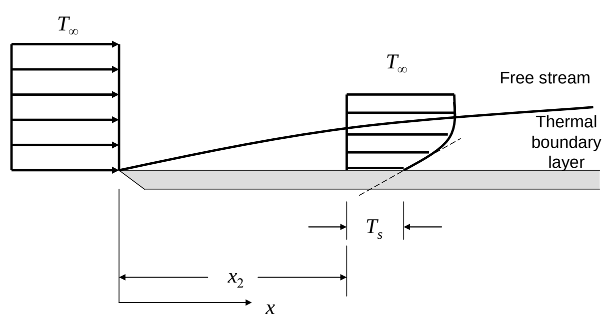
PROBLEM 01 (Cont.)

The magnitude of the heat flux is proportional to the temperature gradient at the surface, $\partial T / \partial y\big|_{y=0}$, which is shown schematically as a dashed line. The temperature gradient is steeper (larger) at x_1 where the thermal boundary layer is thinner and less steep (smaller) at x_3 where the thermal boundary layer is thicker. Therefore, the magnitude of the local heat flux is largest at x_1 and smallest at x_3. <

(c) As the free stream velocity increases the boundary layer becomes thinner. Sketches for a low and high free stream velocity are shown below.



Low freestream velocity case.



High freestream velocity case.

The temperature gradient, shown as the dashed line, is steeper for the higher free stream velocity case. Therefore the higher free stream velocity case has the higher convective heat flux. <

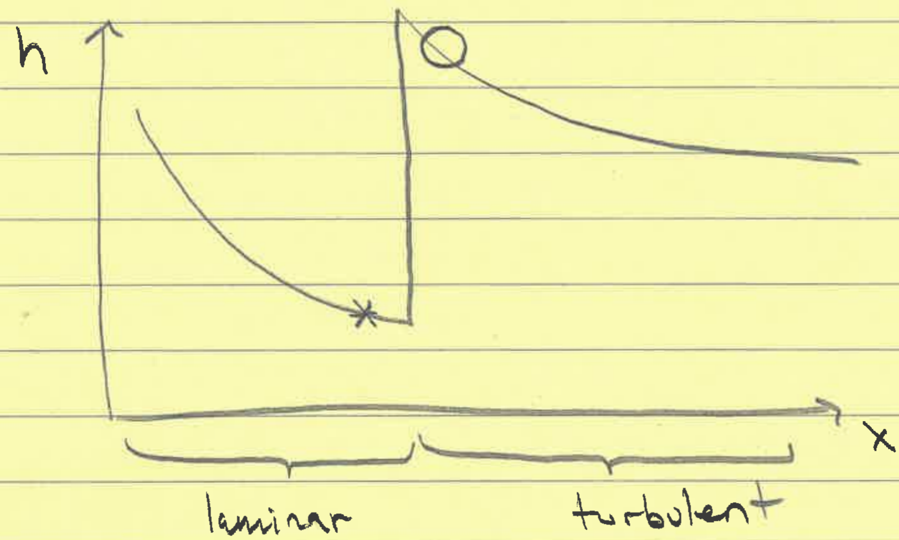
COMMENTS: It is important to understand how the temperature gradient at the surface varies as the thickness of the boundary layer changes.

② Is $Nu|_{x=L} > \overline{Nu}$?

same as

$h_{x=L} > \overline{h}$?

$$Nu = \frac{hL_c}{k_f}$$



→ depends on length of plate

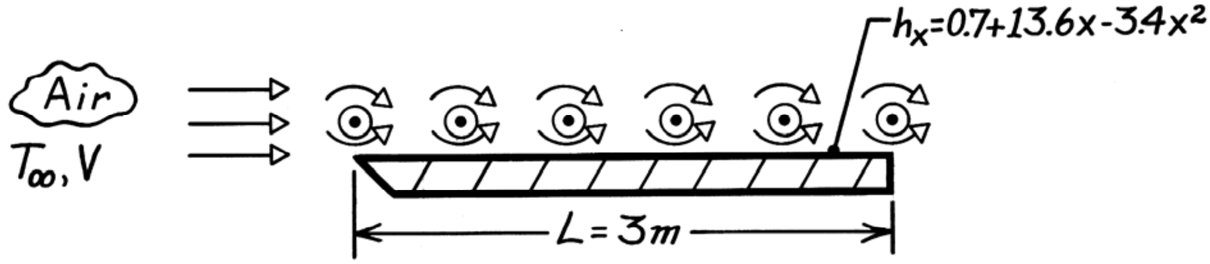
If only laminar flow *
then $\overline{Nu} > Nu_{x=L}$
since $\overline{h} > h_{x=L}$

If end of plate has trancition O
then $\overline{Nu} < Nu_{x=L}$

PROBLEM 03

KNOWN: Distribution of local convection coefficient for obstructed parallel flow over a flat plate.

FIND: Average heat transfer coefficient and ratio of average to local at the trailing edge.
SCHEMATIC:



ANALYSIS: The average convection coefficient is

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\int_0^L \left(0.7 + 13.6x - 3.4x^2\right) dx$$

$$\bar{h}_L = \frac{1}{L}\left(0.7L + 6.8L^2 - 1.13L^3\right) = 0.7 + 6.8L - 1.13L^2$$

$$\bar{h}_L = 0.7 + 6.8(3) - 1.13(9) = 10.9 \text{ W/m}^2 \cdot \text{K}. \qquad <$$

The local coefficient at x = 3m is

$$h_L = 0.7 + 13.6(3) - 3.4(9) = 10.9 \text{ W/m}^2 \cdot \text{K}.$$

Hence,

$$\bar{h}_L / h_L = 1.0. \qquad <$$

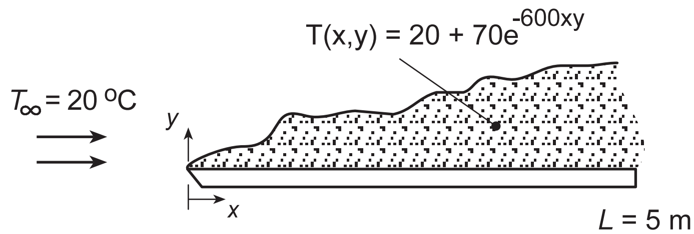
COMMENTS: The result $\bar{h}_L / h_L = 1.0$ is unique to x = 3m and is a consequence of the existence of a maximum for $h_x(x)$. The maximum occurs at x = 2m, where $(dh_x / dx) = 0$ and $(d^2h_x / dx^2 < 0.)$

PROBLEM 04

KNOWN: Temperature distribution in boundary layer for air flow over a flat plate.

FIND: Variation of local convection coefficient along the plate and value of average coefficient.

SCHEMATIC:



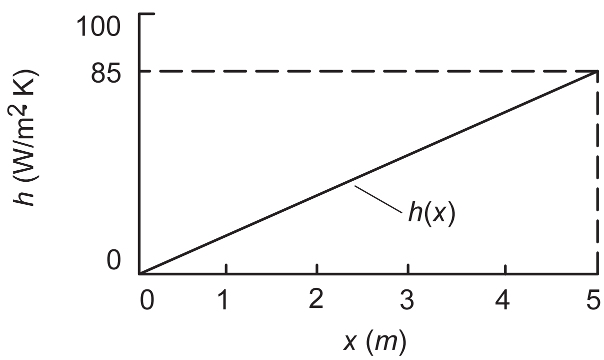
ANALYSIS: From Eq. 6.5,

$$h = -\frac{k\,\partial T/\partial y|_{y=0}}{(T_s - T_\infty)} = +\frac{k(70\times 600x)}{(T_s - T_\infty)}$$

where $T_s = T(x,0) = 90°C$. Evaluating k at the arithmetic mean of the freestream and surface temperatures, $\bar{T}$ = (20 + 90)°C/2 = 55°C = 328 K, Table A.4 yields k = 0.0284 W/m·K. Hence, with $T_s - T_\infty$ = 70°C = 70 K,

$$h = \frac{0.0284\,\text{W/m}\cdot\text{K}\,(42{,}000x)\,\text{K/m}}{70\,\text{K}} = 17x\left(\text{W}/\text{m}^2\cdot\text{K}\right) \qquad <$$

and the convection coefficient increases linearly with x.



The average coefficient over the range 0 ≤ x ≤ 5 m is

$$\bar{h} = \frac{1}{L}\int_0^L h\,dx = \frac{17}{5}\int_0^5 x\,dx = \frac{17}{5}\left.\frac{x^2}{2}\right|_0^5 = 42.5\,\text{W}/\text{m}^2\cdot\text{K} \qquad <$$